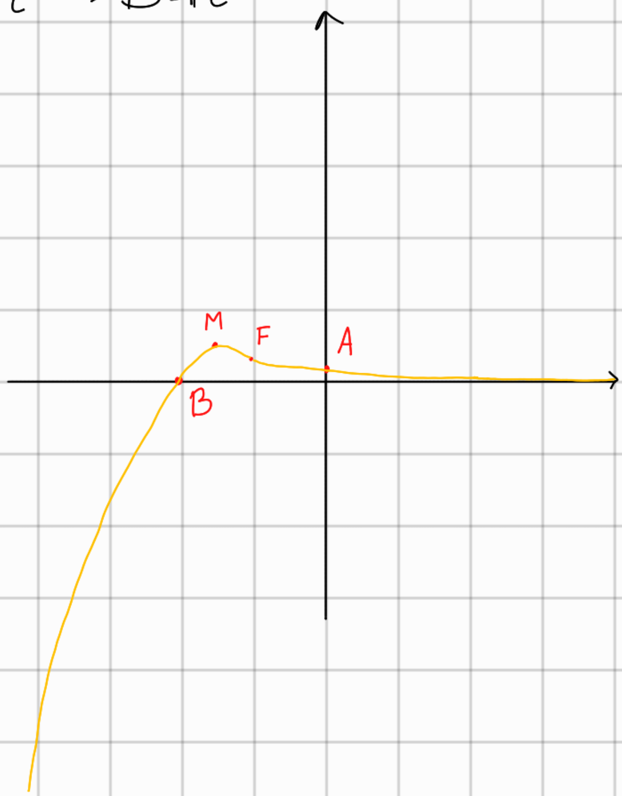
ESAME DI MATEMATICA E STATISTICA 09/04/2024

ESERCIZIO 1

$$f(x) = \frac{x+2}{e^{2x+3}}$$

- C.E: $e^{2x+3} \neq 0 \quad \forall x \in \mathbb{R} \Rightarrow D = \mathbb{R}$



- INTERSEZIONE ASSE y

$$f(0) = \frac{2}{e^3} \simeq 0,1 \qquad A = \left(0, \frac{2}{e^3}\right)$$

- STUDIO SEGNO E INTERSEZIONE ASSE x

$$\frac{x+2}{e^{2x+3}} \geq 0$$

$x+2 \geq 0$ | $e^{2x+3} > 0$

$x \geq -2$ | $\forall x \in \mathbb{R}$

$-$ $+$

-2

$f(x)$ è negativa $x \in (-\infty, -2)$

$f(x)$ è positiva $x \in (-2, +\infty)$

$f(x)$ si annulla $x = -2 \longrightarrow B(-2, 0)$

- LIMITI

$$\lim_{x \to -\infty} \frac{x+2}{e^{2x+3}} = -\infty \qquad (x+2 \to -\infty,\ e^{2x+3} \to 0^+)$$

$$\lim_{x \to +\infty} \frac{x+2}{e^{2x+3}} = 0 \qquad (x+2 \to +\infty,\ e^{2x+3} \to +\infty)$$

→ ASINTOTO ORIZZONTALE DESTRO

F.I per confronto infiniti

- DERIVATA PRIMA

$$f'(x) = \frac{\cancel{e^{2x+3}} - (x+2)2\cancel{e^{2x+3}}}{(e^{2x+3})^{\cancel{2}}} = \frac{1-2x-4}{e^{2x+3}} = \frac{-2x-3}{e^{2x+3}}$$

$f'(x) \geq 0$

$-2x-3 \geq 0$ | $e^{2x+3} > 0$

$-2x \geq 3$ | $\mathbb{R}$

$x \leq -\frac{3}{2}$

$+$ $-$

$-\frac{3}{2}$

$f(x)$ crescente per $x \in (-\infty, -\frac{3}{2})$

$f(x)$ decrescente per $x \in (-\frac{3}{2}, +\infty)$

$f(x)$ stazionaria per $x = -\frac{3}{2}$ $\quad M = \left(-\frac{3}{2}, \frac{1}{2}\right)$ P.TO DI MASSIMO

$$f\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}+2}{e^{-\frac{3}{2}2+3}} = \frac{\frac{1}{2}}{e^0} = \frac{1}{2}$$

■ DERIVATA SECONDA

$$f''(x) = \frac{-2\cancel{e^{2x+3}} - (-2x-3)2\cancel{e^{2x+3}}}{(e^{2x+3})^{\cancel{2}}} = \frac{-2+4x+6}{e^{2x+3}} = \frac{4x+4}{e^{2x+3}}$$

$f''(x) \geq 0$

$4x+4 \geq 0$ | $e^{2x+3} > 0$

$4x \geq -4$ | $\mathbb{R}$

$x \geq -1$

$f(x)$ concava per $x \in (-\infty, -1)$

$f(x)$ convessa per $x \in (-1, +\infty)$

$f(x)$ presenta un p.to di flesso $x = -1$ $\quad F\left(-1, \frac{1}{e}\right)$

$$f(-1) = \frac{-1+2}{e^{-2+3}} = \frac{1}{e} \simeq 0.36$$

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b. CALCOLO $\quad f'(-1) = \dfrac{2-3}{e^{-2+3}} = -\dfrac{1}{e} \simeq -0.36$

Essendo un valore negativo, in tale punto la funzione risulterà decrescente

c. Data $f: D \to \mathbb{R}$ funzione, $x_0 \in D$, f è derivabile in x_0 se esiste finito il limite del rapporto incrementale $\displaystyle\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$

La funzione valore assoluto $f(x) = |x|$ non è derivabile in $x_0 = 0$

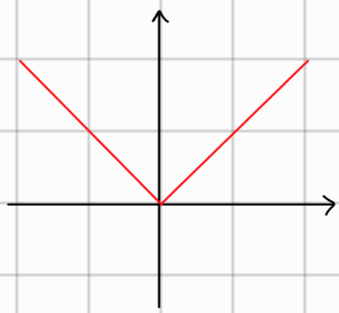
$$\lim_{h \to 0^+} |x| = 1 \neq \lim_{h \to 0^-} |x| = -1$$

$\Downarrow$

$\displaystyle\lim_{h \to 0} |x|$ NON ESISTE.

$\Downarrow$

f non è derivabile



ESERCIZIO 2

$$f(x)=\left(x-\frac{\pi}{2}\right)\cos(2x)$$

Le primitive di f sono le funzioni $F(x)$ $\forall c\in\mathbb{R}$

a. $$F(x)=\int\underbrace{\left(x-\frac{\pi}{2}\right)}_{f}\underbrace{\cos(2x)}_{g'}dx=\frac{1}{2}\left(x-\frac{\pi}{2}\right)\sin(2x)-\int\frac{1}{2}\sin(2x)dx=\frac{1}{2}\left(x-\frac{\pi}{2}\right)\sin(2x)+\frac{1}{4}\cos(2x)+c$$

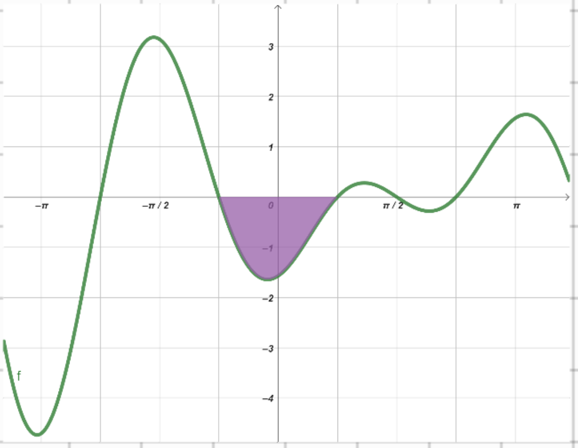
$f \to f'(x)=1$ $\qquad$ $g' \to g=\frac{1}{2}\sin(2x)$

b. $$\int_{-\pi/4}^{\pi/4}(x-1)\cos(2x)dx=\frac{1}{2}\left(x-\frac{\pi}{2}\right)\sin(2x)+\frac{1}{4}\cos(2x)\Big|_{-\pi/4}^{\pi/4}=$$

$$=-\frac{1}{2}\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1}+\frac{1}{4}\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}-\left[-\frac{1}{2}\left(-\frac{\pi}{4}-\frac{\pi}{2}\right)\underbrace{\sin\left(-\frac{\pi}{2}\right)}_{-1}+\frac{1}{4}\underbrace{\cos\left(-\frac{\pi}{2}\right)}_{0}\right]=$$

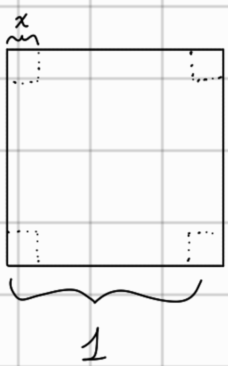
$$=-\frac{1}{2}\left(-\frac{\pi}{4}\right)+\frac{1}{2}\left(-\frac{3}{4}\pi\right)=-\frac{\pi}{8}-\frac{3}{8}\pi=-\frac{4}{8}\pi=-\frac{\pi}{2}$$

c.



ESERCIZIO 3

$$V(x)=x(1-2x)^2$$

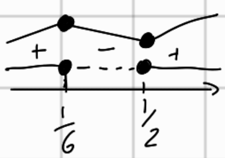


$$V(x)=x(1-4x+4x^2)=4x^3-4x^2+x$$

$$V'(x)=12x^2-8x+1\geq 0$$

$$\Delta=64-48=16$$

$$x_{12}=\frac{8\pm\sqrt{16}}{24}\quad\begin{cases}x_1=\frac{8-4}{24}=\frac{1}{6}\\ x_2=\frac{8+4}{24}=\frac{1}{2}\end{cases}$$



$$x\leq\frac{1}{6}\quad\vee\quad x\geq\frac{1}{2}$$

$V(x)$ è massimo per $x=\frac{1}{6}$

ESERCIZIO 4

ANNO	X	1928	1936	1948	1952	1956	1960	1968	1972	1976
TEMPO (s)	Y	302	285	281	271	267	258	249	240	232

$$\bar{y} = \frac{302 + 288 + \cdots + 238}{9} = \frac{2385}{9} = 265$$

mediana $(y) = 267$ (I DATI SONO DISPARI E GIA' RIORDINATI)

$$\sigma_y^2 = \frac{1}{9}\left[(302-265)^2 + (285-265)^2 + \cdots + (232-265)^2\right] = \frac{1}{9}\left[37^2 + 20^2 + \cdots + (-33)^2\right] = \frac{1}{9}\left[1369 + 400 + \cdots + 1089\right] = \frac{4084}{9} = 453$$

b. $$\bar{x} = \frac{1928 + 1936 + \cdots + 1976}{9} = \frac{17596}{9} \simeq 1955.11$$

$$\sigma_{xy} = \frac{1}{9}\left[(1928-1955.11)(302-265) + (1936-1955.11)(285-265) + \cdots + (1976-1955.11)(232-265)\right]$$

$$= \frac{1}{9}\left[(-27.11)(37) + (-19.11)(20) + \cdots + (20.89)(-33)\right] = \frac{1}{9}\left[-1003.11 - 382.11 - \cdots - 689.33\right]$$

$$= -\frac{2868}{9} = -318.67$$

$$\sigma_x^2 = \frac{1}{9}\left[(-27.11)^2 + (-19.11)^2 + \cdots + (20.89)^2\right] = \frac{1}{9}\left[735.01 + 365.23 + \cdots + 436.34\right] = \frac{1}{9}\left[2072.89\right] = 230.321$$

$$\alpha = \frac{\sigma_{xy}}{\sigma_x^2} = -\frac{318.67}{230.321} \simeq -1.384$$

$$\beta = \bar{y} - \alpha\bar{x} = 265 + (1.384)(1955.11) \simeq 2970$$

$$\text{TEMPO}(2024) = \alpha \cdot 2024 + \beta = (-1.384)(2024) + 2970 \simeq 170 \text{ s} = 2'50''$$